Week 3 – Wednesday
Mathematical Modeling (Math 420/620)

Paul J. Hurtado

9 Sept, 2015
The Division of Applied Mathematics at Brown University invites undergraduate students to attend a workshop on Integrating Dynamics and Stochastics on November 13, 2015. The goal of the workshop is to inform advanced undergraduate students who are interested in pursuing graduate studies in research areas at the intersection of dynamics and stochastics. The workshop will feature talks by faculty, postdocs and graduate students working in various fields related to dynamics and stochastics. There will also be opportunities for attendees to interact with faculty, postdocs and students in the division. The workshop will run from 9am to 5pm with social activities following the workshop.

To apply, please see the following mathprograms.org page:

http://www.mathprograms.org/db/programs/384

For additional information, see:

http://www.dam.brown.edu/people/lipshutz/workshop2015.html
Projects

Two Approaches:

A. Reproduce key results and figures in a published paper, modify and repeat to address a related question.

Finding a paper: UNR Library, www.webofknowledge.com, etc.
Projects

Two Approaches:

A. Reproduce key results and figures in a published paper, modify and repeat to address a related question.

Finding a paper: UNR Library, www.webofknowledge.com, etc.

B. Original research – come talk to me ASAP!
Projects

Two Approaches:

A. Reproduce key results and figures in a published paper, modify and repeat to address a related question.
   Finding a paper: UNR Library, www.webofknowledge.com, etc.
B. Original research – come talk to me ASAP!

Q: How involved should be be?
A: Not trivially easy, not a full publication (perhaps a start of one?).

Q: Can we work in groups?
A: Yes, but only on closely relate independent projects.
Q: ??
Projects

Two Approaches:

A. Reproduce key results and figures in a published paper, modify and repeat to address a related question.
   Finding a paper: UNR Library, www.webofknowledge.com, etc.
B. Original research – come talk to me ASAP!

Q: How involved should be be?
A: Not trivially easy, not a full publication (perhaps a start of one?).

Q: Can we work in groups?
A: Yes, but only on closely relate independent projects.
Q: ??

Update on your project status due on Monday.
Which journals? Which kinds of questions?
Example 1.1

Q: When should the pig be sold to maximize profit?

Profit $P$ is revenue $(R)$ minus cost $(C)$, therefore

\[
P(t) = R(t) - C(t) \\
= p \cdot w - c \cdot t \\
= (0.65 - 0.01 t)(200 + 5 t) - 0.45 t
\]
Example 1.1: Generalized Model

Profit Equation:

\[ P(t) = pw - ct = (p(t)(p_0 - rt))(w(t)(w_0 + gt)) - ct \]

Variables (time dependent):
- \( p \) - pig price per pound
- \( w \) - weight (lbs)
- \( t \) - time (days)

Parameters (constants):
- \( p_0 \) - initial pig price per pound
- \( r \) - daily decrease in price \( p \)
- \( w_0 \) - initial weight of pig
- \( g \) - daily growth rate of pig
- \( c \) - cost to keep pig ($/day)
## R code to plot $P(t)$ and $P'(t)$

```r
x = seq(0, 20, length = 200)
Pt = expression((0.65 - 0.01*x)*(200 + 5*x) - 0.45*x, 'x')
dPt = D(Pt, 'x')
plot(x, (0.65 - 0.01*x)*(200 + 5*x) - 0.45*x, type = "l", lwd = 2,
     ylab = "Profit $P(t)$", xlab = "Time (days)"); abline(v = 8, col = "red")
plot(x, eval(dPt), type = "l", lwd = 2, ylab = "Profit Rate $P'(t)$", xlab = "Time (days)")
x0pt = 8;
abline(h = 0); abline(v = x0pt, col = "red")
```

![Graph of $P(t)$](image1.png)

![Graph of $P'(t)$](image2.png)
```
# R code to plot P(t) and P'(t)
x = seq(0, 20, length = 200)
P = function(x) {
  (0.65 - 0.01*x)*(200 + 5*x) - 0.45*x
}
dP = function(x) {
  (8-x)/10
}; # Use Maxima!
plot(x, P(x), type="l", lwd=2,
    ylab="Profit P(t)", xlab="Time (days)"); abline(v=8, col="red")
plot(x, dP(x), type="l", lwd=2, ylab="Profit Rate P'(t)", xlab="Time (days)"
)xOpt = 8;
abline(h=0); abline(v=xOpt, col="red")
```
## R code to plot \( P(t) \) and \( P'(t) \)

```r
x = seq(0, 20, length=200)
P = function(x) {
  (0.65-0.01*x)*(200+5*x)-0.45*x
}
dP = function(x) {(8-x)/10}  # Use Maxima!

plot(x, P(x), type="l", lwd=2,
     ylab="Profit \( P(t) \)", xlab="Time (days)"); abline(v=8, col="red")

plot(x, dP(x), type="l", lwd=2, ylab="Profit Rate \( P'(t) \)" , xlab="Time (days)"

xOpt = optimize(P,c(0,20),maximum=TRUE)  # 1D optimization in R
abline(h=0); abline(v=xOpt, col="red")
```

![Graphs showing \( P(t) \) and \( P'(t) \)](image)
Multivariable Optimization: Ex 2.1 (Pg 21)

Q: How many 19- & 21-inch TVs maximize profit?

Profit Equation:

\[ P(x_{19}, x_{21}) = p x_{19} + q x_{21} - C = (p_{0} - p_{19} x_{19} - p_{21} x_{21}) x_{19} + (q_{0} - q_{19} x_{19} - q_{21} x_{21}) x_{21} - (c_{0} + c_{19} x_{19} + c_{21} x_{21}) \]

Variables (time dependent):
- \( x_{19} \): number of 19in TVs sold
- \( x_{21} \): number of 21in TVs sold

Parameters (constants):
- \( p_{0} \): Retail price of 19-inch TV
- \( q_{0} \): Retail price of 19-inch TV
- \( p_{19} \): 19in discount / 19in TV sold
- \( p_{21} \): 19in discount / 21in TV sold
- \( q_{19} \): 21in discount / 19in TV sold
- ...
Multivariable Optimization: Ex 2.1 (Pg 21)

**Approach #1:** Find “interior” optima by solving $\nabla P = 0$. 

Using Maxima, we can find and solve

\[
\frac{\partial P}{\partial x_19} = p_0 - 2p_{19}x_{19} - (q_{19} + p_{21})x_{21} - c_{19} = 0
\]

\[
\frac{\partial P}{\partial x_{21}} = q_0 - (q_{19} + p_{21})x_{19} - 2q_{21}x_{21} - c_{21} = 0
\]

Solving and using the given parameter values yields $x_{19} = 4735.04...$ and $x_{21} = 7042.74...$.
Multivariable Optimization: Ex 2.1 (Pg 21)

**Approach #1**: Find “interior” optima by solving $\nabla P = 0$.

Using Maxima, we can find and solve

\[
\frac{\partial P}{\partial x_{19}} = p_0 - 2p_{19}x_{19} - (q_{19} + p_{21})x_{21} - c_{19} = 0
\]

\[
\frac{\partial P}{\partial x_{21}} = q_0 - (q_{19} + p_{21})x_{19} - 2q_{21}x_{21} - c_{21} = 0
\]
Multivariable Optimization: Ex 2.1 (Pg 21)

**Approach #1**: Find “interior” optima by solving $\nabla P = 0$.

Using Maxima, we can find and solve

\[
\frac{\partial P}{\partial x_{19}} = p_0 - 2p_{19}x_{19} - (q_{19} + p_{21})x_{21} - c_{19} = 0
\]

\[
\frac{\partial P}{\partial x_{21}} = q_0 - (q_{19} + p_{21})x_{19} - 2q_{21}x_{21} - c_{21} = 0
\]

Solving and using the given parameter values yields

\[x_{19} = 4735.04\ldots \quad x_{21} = 7042.74\ldots\]
Approach #2: Use generic optimization routines to computationally maximize $P(x_{19}, x_{21})$ over $x_{19}, x_{21} > 0$.

See Ch2-optimization.R