Dynamical Systems
Week 5 – Wednesday

Mathematical Modeling (Math 420/620)

Paul J. Hurtado

23 Sept, 2015

Colloquium

The four-genus of connected sums of torus knots

Cornelia Van Cott, UCSF

2:30-3:30pm, **Thursday** (9/24) in **SEM 234**

Abstract: Every knot bounds infinitely many different surfaces in B^4. In the special case of torus knots and their connected sum, we consider the problem of finding the simplest possible such surface. This problem has a long history of results, dating back to the work of Fox and Milnor and continuing to the present day. We will show that the classical signature function as well as the newly defined Upsilon function both provide some elegant answers to the problem, yet some unknowns remain.

www.unr.edu/math/colloquium

MATLAB® & SIMULINK®

MathWorks Day Seminar

University of Nevada Reno

September 29, 2015

Join MathWorks engineers as they provide insight into the latest features of the MATLAB and Simulink product families.

Tuesday, Sept 29th
10:00 a.m. – 3:00 p.m.

Registration will begin 15 minutes before the session begins. Walk-ins are welcome.

Session will take place:

EEL-200

(Earthquake Engineering Lab Auditorium)

MATLAB & Simulink Seminars:

- **Session 1:** (10:00 a.m. – 12:00 p.m.) **Mathematical Modeling with MATLAB**
- **Session 2:** (1:00 p.m. – 3:00 p.m.) **Programming Low-Cost Hardware with MATLAB and Simulink**

For more information, contact:
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To register in advance:

www.mathworks.com/UNR/Sept2015

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MATLAB Seminar at University of Nevada Reno

Session One

Date: September 29, 2015
Location: EEL-200 (Earthquake Engineering Lab Auditorium)
Time: 10:00 a.m. - 12:00 p.m.
Registration and sign-in begins at 9:45 a.m. Walk-ins are welcome.
Overview: **Mathematical Modeling with MATLAB**

Mathematical models are critical to understanding and accurately simulating the behavior of complex systems. They enable important tasks such as forecasting system behavior for various "what if" scenarios, characterizing system response, and designing control systems.

In this session, we will demonstrate how you can use MATLAB products for mathematical modeling tasks.

Highlights include:

- Developing models using data fitting and first-principle modeling techniques
- Optimizing the accuracy of mathematical models
- Simulating models and post-processing the results
- Documenting and sharing models

You will also learn about different approaches you can use to develop models, including developing models programmatically using the MATLAB language, deriving closed-form analytical equations using symbolic computation, and leveraging prebuilt graphical tools for specific modeling tasks such as curve and surface fitting.

Session Two

Date: September 29, 2015
Time: 1:00 p.m. – 3:00 p.m.
Location: EEL-200 (Earthquake Engineering Lab Auditorium)
Registration and sign-in begins at 12:45 p.m. Walk-ins are welcome.
Overview: **Programming Low-Cost Hardware with MATLAB and Simulink**

Using MATLAB and Simulink to target low-cost hardware can help you to go from theory to practice, and easily experiment with concepts in mechatronics, robotics, circuit design, programming, controls, signal, image and video processing. MATLAB and Simulink are industry-standard environments for data analysis, multi-domain modeling, simulation and design. They provide a high level, interactive environments, and powerful & customizable built-in functionalities that enable you to analyze, design, simulate, implement, and test a variety of systems.

In this session, we will demonstrate how to design, simulate, and deploy algorithms from Simulink to a Raspberry Pi hardware.

Highlights include:

- Designing and simulating image processing algorithms in MATLAB and Simulink
- Embedding control logic in low-cost hardware such as Arduino, LEGO MINDSTORMS and Raspberry Pi without writing any C-code
- Monitoring sensor signals and tuning system parameters in real time

No prior experience with MATLAB or Simulink is necessary.

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Overview

Building Dynamic Models, ODEs

- Mean Field Equations
- "Bathtub" Models

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Building Dynamic Models, ODEs

- Mean Field Equations
- "Bathtub" Models

Analysis of Dynamic Models

- State Space & Vector Fields
- Asymptotic Behavior
- Attractors
- Equilibrium Stability Analysis

Mean Field Equations

Example: Suppose there are N_0 atoms of radioactive ${}^{238}_{92}\mathrm{U}$. Over time interval Δt each can decay w.p. $\lambda \Delta t$.

Let $N(t)$ be the number of uranium atoms. The number lost during time interval $[t, t + \Delta t]$ is approximately a binomial *random variable* with parameters $n = N(t)$ and $p = \lambda \Delta t$. Thus, the *expected number* lost is $n\,p = \lambda\,N(t)\,\Delta t$.

Assuming N_0 is large, then $N(t + \Delta t) - N(t) \approx -\lambda\,N(t)\,\Delta t$. Taking $\Delta t \to 0$ we can derive the **mean field** model:

$$\frac{dN(t)}{dt} = -\lambda N(t), \qquad N(0) = N_0$$

Mean Field Equations

Example: Suppose there are U_0 atoms of radioactive ${}^{238}_{92}\text{U}$. Over time interval Δt each can decay w.p. $\lambda_\alpha \Delta t$ to ${}^{234}_{90}\text{Th}$ and α particle ${}^{4}_{2}\text{He}$. Thorium-234 can then decay via loss of a β particle (positron) to protactinium-234 w.p. $\lambda_\beta \Delta t$.

Let $T(t)$ be the number of thorium atoms, and $P(t)$ the number of protactinium atoms. We can now use the model

$$
\begin{aligned}
\frac{dU(t)}{dt} &= -\lambda_\alpha U(t) \\
\frac{dT(t)}{dt} &= \lambda_\alpha U(t) - \lambda_\beta T(t) \\
\frac{dP(t)}{dt} &= \lambda_\beta T(t)
\end{aligned}
$$

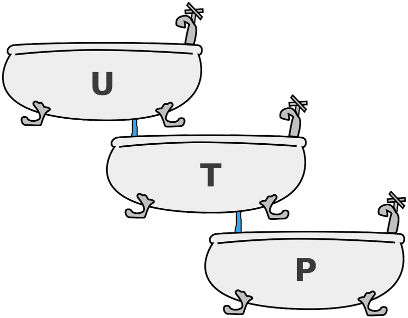
ODEs: "Bathtub" Models

Model the "flow" of mass from one compartment to another:

$$\frac{dU(t)}{dt} = -\lambda_\alpha U(t)$$

$$\frac{dT(t)}{dt} = \lambda_\alpha U(t) - \lambda_\beta T(t)$$

$$\frac{dP(t)}{dt} = \lambda_\beta T(t)$$



Intuition for ODE model terms

- **Recall the 5-step process!**

  Question? Assumptions? Simplify, etc...

- ODE models often *average* over heterogeneity, space, etc.

- **Linear terms** correspond to **exponential decay** rates.

- More complex transition rates? Derive[1] terms accordingly.

[1] Remember your credo: Lie, Cheat, Steal! (see Ch. 9 in Ellner & Guckenheimer)

Dynamic Model (ODE) Basics

Suppose $\mathbf{x} \in \mathbb{R}^n$, functions $f = [f_1, f_2, \ldots, f_n]$ are *smooth*[2], and

$$\frac{d\mathbf{x}}{dt} = f(\mathbf{x}), \qquad \mathbf{x}(0) = \mathbf{x_0}.$$

[2] Continuous partial derivatives near $\mathbf{x_0}$ guarantee existence, uniqueness of solutions.

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State Variables:	$\mathbf{x} = [x_1, x_2, \ldots, x_n]$
Initial Conditions:	$\mathbf{x_0}$
State Space:	$S \subseteq \mathbb{R}^n$
Vector Field:	f
Parameter Space:	(Ex) $\mathbb{R}^{n^2}$ if f is a full linear system.
Trajectory/Orbit:	Solutions $\mathbf{x}(t)$ to the above IVP.

[2] Continuous partial derivatives near $\mathbf{x_0}$ guarantee existence, uniqueness of solutions.

Examples

What are the state variables? State space? Parameter space?

1.

$$\frac{dx}{dt} = r\,x\,(1 - x/K)$$

2.

$$\frac{dN}{dt} = r\,N\left(1 - (N/K)^{\theta}\right)$$

3.

$$\frac{du}{d\tau} = u\,(1 - u^{\theta})$$

4.

$$\begin{aligned}\dot{H} &= r_H\,H - a_H\,H^2 - b_H\,S\,H\\ \dot{S} &= r_S\,S - a_S\,S^2 - b_S\,H\,S\end{aligned}$$