

1. Use the R code provided in class (on the website) to find the input values that maximize the following functions. Do this using two methods: First, find the values that maximize  $f$  *analytically* (i.e., using calculus) then do it *computationally* (e.g., using `optimize` or `optimx` in R). How well the computationally and analytically derived values agree?

- a. Let  $a = 1$ ,  $b = 2$ , and  $c = 0.5$ . For  $x > 0$ , maximize

$$f(x) = \frac{x}{a + bx + cx^2}$$

- b. Maximize  $f$  over all  $x, y \in \mathbb{R}$ , where

$$f(x, y) = e^{-x^2 - 4y^2 + 4x - 4}$$

- c. Minimize  $f$  over all  $w, x, y, z \in \mathbb{R}$ , where

$$\sqrt{17z^2 + \pi y^2 + \sqrt{2}x^2 + ew^2 + 1}$$

2. Use LaGrange Multipliers (see section 2.2) to maximize  $x + y$  when  $x$  and  $y$  are constrained to the circle  $x^2 + y^2 = 4$ .

**Bonus:** Use polar coordinates to solve #2 as an unconstrained optimization problem.