Example Probability Distributions

Name (Discrete)	**S**	**Θ**	**PMF**	**CDF**	**E(X)**	**Var(X)**	**$\mathbf{M_X(t)}$**
Bernoulli	$x \in \{0,1\}$	p	$p^x(1-p)^{1-x}$	-	p	$p(1-p)$	$1-p+p\,e^t$
Binomial	$\{0,...,n\}$	$n,\, p$	$\binom{n}{x}p^x(1-p)^{n-x}$	-	np	$np(1-p)$	$(1-p+p\,e^t)^n$
Multinomial	$\sum X_i = n$	$n, \sum p_i = 1$	$\frac{n!}{x_1!\cdots x_k!}p_1^{x_1}\cdots p_k^{x_k}$	-	$E(X_i)=np_i$	$V(X_i)=np_i(1-p_i)$	$\left(\sum_{i=1}^k p_i e^{t_i}\right)^n$
Hypergeometric	$\{0,...,n\}$	$n,\, w,\, N$	$\binom{w}{x}\binom{N-w}{n-x}/\binom{N}{n}$	-	$n\,(w/N)$	$n(w/N)(1-w/N)\,\frac{N-n}{N-1}$	-
Gen. Hypergeometric	$\{0,...,n\}^k$	$N=\sum n_i,\, n$	$\binom{n_1}{x_1}\cdots\binom{n_k}{x_k}/\binom{N}{n}$	-	Marginals are Hypergeometric		-
Geometric (# failures)	$\{0,1,...\}$	p	$(1-p)^x p$	$1-(1-p)^{k+1}$	$(1-p)/p$	$(1-p)/p^2$	$\frac{p}{1-(1-p)\exp(t)}$
Negative Binomial	$\{0,1,...\}$	$n,\, p$	$\binom{x+n-1}{x}p^n(1-p)^x$	-	$n(1-p)/p$	$n(1-p)/p^2$	$\left(\frac{p}{1-(1-p)\exp(t)}\right)^n$
Negative Binomial	$\{0,1,...\}$	$n,\, \mu$	$\frac{\Gamma(n+x)}{\Gamma(n)x!}(\frac{n}{n+\mu})^n(\frac{\mu}{n+\mu})^x$	-	μ	$\mu+\mu^2/n$	$\left(\frac{n}{n+\mu-\mu\, e^t}\right)^n$
Geometric (# trials)	$\{1,2,...\}$	p	$(1-p)^{x-1}p$	$1-(1-p)^k$	$1/p$	$(1-p)/p^2$	$\frac{p\exp(t)}{1-(1-p)\exp(t)}, \quad t<\ln\left(\frac{1}{1-p}\right)$
Negative Binomial	$\{n,...\}$	$n,\, p$	$\binom{x-1}{n-1}p^n(1-p)^{x-n}$	-	n/p	$n(1-p)/p^2$	$(\frac{p\exp(t)}{1-(1-p)\exp(t)})^n,\ t<\ln\left(\frac{1}{1-p}\right)$
Poisson(rT)	$\{0,1,...\}$	$r,\, T$	$e^{-rT}\frac{(rT)^x}{x!}$	-	rT	rT	$\exp(rT(e^t-1))$
Poisson(λ)	$\{0,1,...\}$	λ	$e^{-\lambda}\frac{\lambda^x}{x!}$	-	λ	λ	$\exp(\lambda(e^t-1))$

Name (Continuous)	**S**	**Θ**	**PDF**	**CDF**	**E(X)**	**Var(X)**	**$\mathbf{M_X(t)}$**
Uniform	$[a,b]$	$a,\, b$	$\frac{1}{b-a}$	$\frac{x-a}{b-a}$	$\frac{a+b}{2}$	$\frac{1}{12}(b-a)^2$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$
Exponential (rate r)	$[0,\infty)$	r	$r\,e^{-rx}$	$1-e^{-rx}$	$\frac{1}{r}$	$\frac{1}{r^2}$	$(1-t/r)^{-1}$
Exponential (mean θ)	$[0,\infty)$	θ	$1/\theta\,e^{-x/\theta}$	$1-e^{-x/\theta}$	θ	θ^2	$(1-\theta t)^{-1}$
Gamma(shape k, rate r)	$[0,\infty)$	$k,\, r$	$\frac{r^k}{\Gamma(k)}x^{k-1}e^{-rx}$	-	$\frac{k}{r}$	$\frac{k}{r^2}$	$(1-t/r)^{-k}$
Gamma(shape k, scale θ)	$[0,\infty)$	$k,\, \theta$	$\frac{\theta^{-k}}{\Gamma(k)}x^{k-1}e^{-x/\theta}$	-	$k\theta$	$k\theta^2$	$(1-\theta t)^{-k}$
Gamma(shape α, rate β)	$[0,\infty)$	$\alpha,\, \beta$	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}$	-	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$	$(1-t/\beta)^{-\alpha}$
Normal	$\mathbb{R}$	$\mu,\, \sigma$	$\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$	-	μ	σ^2	$\exp(\mu t+\frac{1}{2}\sigma^2t^2)$
Beta	$[0,1]$	$a,\, b$	$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1}$	-	$\frac{a}{a+b}$	$\frac{ab}{(a+b)^2(a+b+1)}$	-
Pareto	$[x_m,\infty)$	$x_m,\, \alpha$	$\alpha x_m^\alpha/x^{\alpha+1}$	$1-\left(\frac{x_m}{x}\right)^\alpha$	$\frac{\alpha x_m}{\alpha-1}$; ($\infty$ if $\alpha\le 1$)	$\frac{\alpha\, x_m^2}{(\alpha-1)^2(\alpha-2)}$ if $\alpha>2$	-

Note: Quantities not shown include: *median*, *mode*, *quantile function*, *skewness*, *kurtosis*, *entropy*, *characteristic function* (Fourier Transformed PDF), *conjugate prior*.

Generalized Applications

Name (Discrete)	Application
Bernoulli	Binary (i.e., 0 or 1) outcome, with probability p of focal outcome (aka 'success'). That is, $\mathcal{P}(X=1)=p$.
Binomial	The distribution of # successes in n Bernoulli trials. 0 to n successes possible. Ex: Flipping a coin n times, counting "heads".
Multinomial	Like binomial, but for k outcome types, not just 2 outcomes (i.e, 0 or 1). Ex: Distribution of outcomes from rolling a k-sided dice n times.
Hypergeometric	Ex: Distribution of # black balls obtained by drawing (*without* replacement) n balls from an urn containing K black and $N-K$ white balls.
Gen. Hypergeometric	Like Hypergeometric (above), but with multiple ($k>2$) colors of balls. (Hypergeometric is just the $k=2$ case)
Geometric (# failures)	Ex: Number of failures before a success, where probability of success at each trial is p.
Negative Binomial	Like Geometric (above), but the number of failures (not number of trials!) before the n^{th} success. ($n=1$ gives the above distribution.)
Negative Binomial	Same as above (i.e., count of # failures before n^{th} success) but parameterized in terms of the mean μ instead of success probability p.
Geometric (# trials)	Variant of Geometric (see above), but for counting the number of *total trials* (including the successful trial), not just failures.
Negative Binomial	Variant of Negative Binomial above, but counts *total trials* (including successes) taken to reach the n^{th} success (alt. parameterization not shown).
Poisson(rT)	Count of events in interval $[0, T]$ where the inter-event intervals are exponentially distributed with rate r (i.e., exponential with mean $1/r$.)
Poisson(λ)	Same as above, but parameterized in terms of the expected (mean) number of events (λ).

Name (Continuous)	Application
Uniform	All outcomes are equally likely.
Exponential (rate r)	Continuous version Geometric Distribution: Time duration until an event. Over small time interval Δt, the probability of the event is $p \approx r\,\Delta t$.
Exponential (mean θ)	Same as above, but parameterized in terms of the mean duration time $\theta = 1/r$.
Gamma(shape k, rate r)	Continuous version of Negative Binomial: Time until k^{th} event, where probability of event in Δt is $\approx r\,\Delta t$. **Alt:** Sum of k exponentials (rate r).
Gamma(shape k, scale θ)	Same as above, but follows the alternate parameterization of the Exponential using it's mean, not rate. **Alt:** Sum of k exponentials (mean θ).
Gamma(shape α, rate β)	Same as above. A common alternative parameterization that just uses different notation (α and β) for the shape (k) and rate (r), respectively.
Normal	The **Central Limit Theorem** says sums of iid r.v.s are approximately Normal. Ex: Many data that reflect multiple sources of "randomness"
Beta	In Bayesian statistics, its the *conjugate prior distribution* for estimates of p in Bernoulli, binomial, geometric, and negative binomial distributions.
Pareto	A Power Law distribution used to model heavy-tailed data. If X is exponential (rate α), then $Y = x_m\,e^X$ is Pareto.

Updated: 3 December 2015 by Paul J. Hurtado