MATH 461/661 Homework 7

Due Thursday, April 2, 2015 at the beginning of lecture.

In order to receive full credit, you must (1) present the solutions in the order of the assigned problems, (2) write or type legibly, and (3) justify your answers appropriately. For problems with answers given in the back of the text, please remember to **show your work!**

1. **3.5.8**, parts a and d only.
2. **3.5.27**, part a only.
3. **3.6.5**
4. **3.6.8**
5. **3.6.16**
6. See pages 160-161. Let X be uniform on $[a, b]$ with $f(x) = 1/(b-a)$, $a, b > 0$.
   a. Find a formula for the nth moment about the origin.
   b. Find a formula for the nth moment about the mean.
7. Let random variables $X \in [0, \frac{2}{3}]$ and $Y \in [0, 1]$ have joint distribution $f(x, y) = a(xy + y^2)$, where $a > 0$ is a normalizing constant such that $\int_0^{\frac{2}{3}} \int_0^1 f(x, y)dydx = 1$. Recall the definition of conditional probability, $P(A|B) = P(A \cap B)/P(B)$.
   a. **(461)** Show that $a = 3$.
   b. Show that the marginal pdfs are $f_X(x) = \frac{3}{2}x + 1$, and $f_Y(y) = \frac{2}{3}y + 2y^2$.
   c. Show that the conditional pdf for $X|\{Y = \frac{1}{2}\}$ (see the lecture notes, or pg 205) is

$$f_{X|Y=1/2}(x) = \frac{9}{5}x + \frac{9}{10}$$

   c. Show that the conditional pdf for $W = X|\{Y \in [0, \frac{1}{2}]\}$ is

$$f_W(w) = f_{X|Y\in[0,1/2]}(w) = \frac{9}{4}w + \frac{3}{4}.$$

   Hint: Find the pdf by differentiating the cdf,

$$F_W(w) = P(W \le w) = P(X \le w|Y \in [0, 1/2]).$$

   d. **(661)** Show (1) that you can also arrive at the answer in **c** above by assuming

$$f_{X|Y\in A}(w) = \frac{\int_A f(w, y)dy}{\int_A f_Y(y)dy}$$

   and (2) prove whether or not this formula should hold in general.