Example Probability Distributions

Name (Discrete)	**S**	**Θ**	**PMF**	**CDF**	**E(X)**	**Var(X)**	**$M_X(t)$**
Bernoulli	$x \in \{0,1\}$	p	$p^x(1-p)^{1-x}$	-	p	$p(1-p)$	$1-p+p\,e^t$
Binomial	$\{0,...,n\}$	$n,\,p$	$\binom{n}{x}p^x(n-p)^{n-x}$	-	np	$np(1-p)$	$(1-p+p\,e^t)^n$
Multinomial	$\sum X_i = n$	$n,\sum p_i = 1$	$\frac{n!}{x_1!\cdots x_k!}p_1^{x_1}\cdots p_k^{x_k}$	-	$E(X_i) = np_i$	$V(X_i) = np_i(1-p_i)$	$\left(\sum_{i=1}^k p_i e^{t_i}\right)^n$
Hypergeometric	$\{0,...,n\}$	$n,\,w,\,N$	$\binom{w}{x}\binom{N-w}{n-x}/\binom{N}{n}$	-	$n\,(w/N)$	$n(w/N)(1-w/N)\frac{N-n}{N-1}$	-
Gen. Hypergeometric	$\{0,...,n\}^k$	$N = \sum n_i,\,n$	$\binom{n_1}{x_1}\cdots\binom{n_k}{x_k}/\binom{N}{n}$	-	Marginals are Hypergeometric		-
Geometric (# failures)	$\{0,1,...\}$	p	$(1-p)^x p$	$1-(1-p)^{k+1}$	$(1-p)/p$	$(1-p)/p^2$	$\frac{p}{1-(1-p)\exp(t)}$
Negative Binomial	$\{0,1,...\}$	$n,\,p$	$\binom{x+n-1}{x}p^n(1-p)^x$	-	$n(1-p)/p$	$n(1-p)/p^2$	$\left(\frac{p}{1-(1-p)\exp(t)}\right)^n$
Negative Binomial	$\{0,1,...\}$	$n,\,\mu$	$\frac{\Gamma(n+x)}{\Gamma(n)x!}(\frac{n}{n+\mu})^n(\frac{\mu}{n+\mu})^x$	-	μ	$\mu+\mu^2/n$	$\left(\frac{n}{n+\mu-\mu\,e^t}\right)^n$
Geometric (# trials)	$\{1,2,...\}$	p	$(1-p)^{x-1}p$	$1-(1-p)^k$	$1/p$	$(1-p)/p^2$	$\frac{p\exp(t)}{1-(1-p)\exp(t)},\quad t<\ln\left(\frac{1}{1-p}\right)$
Negative Binomial	$\{n,...\}$	$n,\,p$	$\binom{x-1}{n-1}p^n(1-p)^{x-n}$	-	n/p	$n(1-p)/p^2$	$(\frac{p\exp(t)}{1-(1-p)\exp(t)})^n,\,t<\ln\left(\frac{1}{1-p}\right)$
Poisson(λ)	$\{0,1,...\}$	λ	$e^{-\lambda}\frac{\lambda^x}{x!}$	-	λ	λ	$\exp(\lambda(e^t-1))$
Poisson(rT)	$\{0,1,...\}$	$r,\,T$	$e^{-rT}\frac{(rT)^x}{x!}$	-	rT	rT	$\exp(rT(e^t-1))$

Name (Continuous)	**S**	**Θ**	**PDF**	**CDF**	**E(X)**	**Var(X)**	**$M_X(t)$**
Uniform	$[a,b]$	$a,\,b$	$\frac{1}{b-a}$	$\frac{x-a}{b-a}$	$\frac{a+b}{2}$	$\frac{1}{12}(b-a)^2$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$
Exponential (rate r)	$[0,\infty)$	r	$r\,e^{-rx}$	$1-e^{-rx}$	$\frac{1}{r}$	$\frac{1}{r^2}$	$(1-t/r)^{-1}$
Exponential (mean θ)	$[0,\infty)$	θ	$1/\theta\,e^{-x/\theta}$	$1-e^{-x/\theta}$	θ	θ^2	$(1-\theta t)^{-1}$
Gamma(shape k, scale θ)	$[0,\infty)$	$k,\,\theta$	$\frac{\theta^{-k}}{\Gamma(k)}x^{k-1}e^{-x/\theta}$	-	$k\theta$	$k\theta^2$	$(1-\theta t)^{-k}$
Gamma(shape k, rate r)	$[0,\infty)$	$k,\,r$	$\frac{r^k}{\Gamma(k)}x^{k-1}e^{-rx}$	-	$\frac{k}{r}$	$\frac{k}{r^2}$	$(1-t/r)^{-k}$
Gamma(shape α, rate β)	$[0,\infty)$	$\alpha,\,\beta$	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}$	-	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$	$(1-t/\beta)^{-\alpha}$
Normal	$\mathbb{R}$	$\mu,\,\sigma$	$\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$	-	μ	σ^2	$\exp(\mu t+\frac{1}{2}\sigma^2 t^2)$
Beta	$[0,1]$	$a,\,b$	$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1}$	-	$\frac{a}{a+b}$	$\frac{ab}{(a+b)^2(a+b+1)}$	-
Pareto	$[x_m,\infty)$	$x_m,\,\alpha$	$\alpha x_m^\alpha/x^{\alpha+1}$	$1-\left(\frac{x_m}{x}\right)^\alpha$	$\frac{\alpha x_m}{\alpha-1}$; ($\infty$ if $\alpha\le 1$)	$\frac{\alpha\,x_m^2}{(\alpha-1)^2(\alpha-2)}$ if $\alpha>2$	-

Note: Quantities not shown include: *median*, *mode*, *quantile function*, *skewness*, *kurtosis*, *entropy*, *characteristic function* (Fourier Transformed PDF), *conjugate prior*.